# Vector algebra- vector multiplied by scalar

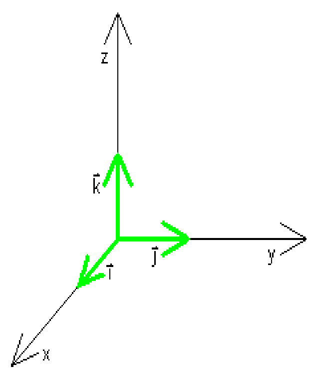
In the sequel we consider only the three dimensional Euclidian vector spaces, denoted by $V_3$.

Based on the usual notations in $\mathbb{R}^3$, a point $P_0$ can be written in Cartesian coordinate form as $P_0 = (x_0, y_0, z_0)$.

We will denote by $\overrightarrow{OP_0}$ the oriented line sequence, the position vector of the point $P_0$, and for which we introduce the similar coordinates $\overrightarrow{OP_0} = \langle x_0, y_0, z_0 \rangle$ .

We will have for two different points $P_1 = (x_1, y_1, z_1)$. and $P_2$ =$(x_2, y_2, z_2)$ in $\mathbb{R}^3$, the oriented line sequence will be denoted by $\overrightarrow{P_1P_2}$, and we will use the coordinate form

$\overrightarrow{P_1P_2} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$, as obviously $\overrightarrow{P_1P_2} = \overrightarrow{OP_2}$ -$\overrightarrow{OP_1}$ . The class of conguence $\left\{ \overrightarrow{P_1P_2} = \langle a, b, c \rangle \mid P_1, P_2 \in \mathbb{R}^3 \right\}$ is by definition the vector $\overrightarrow{v} = \langle a, b, c \rangle \in V_3$. We mention 3 special vectors denoted $\overrightarrow{i} = \langle 1, 0, 0 \rangle$, $\overrightarrow{j} = \langle 0, 1, 0 \rangle$, $\overrightarrow{k} = \langle 0, 0, 1 \rangle$, the unit vectors of the 3 coordinate axis, named coordinate vectors.




## Basic vector operations

We have $\overrightarrow{v} = \langle a, b, c \rangle = a\overrightarrow{i} + b\overrightarrow{j} + c\overrightarrow{k}$.

## Vector multiplied by a scalar

Let us take the field $(\mathbb{R}, +, \cdot)$ and the Abelian group $(V_3, +)$ . We define an external operation type $\mathbb{R} \times V_3 \longrightarrow V_3$, i.e. the operation of multiplying the vector $\overrightarrow{v} = \langle a, b, c \rangle \in V_3$ by the scalar $\lambda$, denoted by $\lambda\overrightarrow{v} = \langle \lambda a, \lambda b, \lambda c \rangle \in V_3$.

**Properties**

The vector $\lambda\overrightarrow{v}$ will be parallel with $\overrightarrow{v}$, except for $\lambda = 0$.

Example. For $\overrightarrow{v} = \langle 2, -1, 3 \rangle$ and $\lambda = 5$ will furnish the vector $5\overrightarrow{v} = \langle 10, -5, 15 \rangle$

We will be able to check this using e.g their vector product, see below.

Further properties:
$\lambda\,(\overrightarrow{v_1}+\overrightarrow{v_2}) = \lambda\overrightarrow{v_1}+\lambda\overrightarrow{v_2}$
$(\lambda_1+\lambda_2)\,\overrightarrow{v} = \lambda_1\,\overrightarrow{v}+\lambda_2\,\overrightarrow{v}$
$\lambda_1\,(\lambda_2\,\overrightarrow{v}) = (\lambda_1\lambda_2)\,\overrightarrow{v}$ and
$\mathit{1}.\overrightarrow{v} = \overrightarrow{v}$, where $\mathit{1}$ is the unit in $\mathbb{R}$.